

Rebuttal Report of Michael Barber, PhD

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1 Introduction

It is particularly frustrating to receive this latest filing from Dr. Chen because he has already had a full opportunity to critique my algorithm during his prior submission, and he chose not to raise any of these concerns at that time. To now introduce them at this late stage—after I have already submitted my own detailed report responding to his methodology—suggests that these are not genuine technical concerns but rather an eleventh-hour attempt to shift focus away from the fundamental shortcomings of his own algorithm that I documented in my October 22 report. Moreover, to the extent these criticisms are offered in earnest, they reveal a basic misunderstanding of how redistricting simulation models actually operate. His commentary betrays a lack of familiarity with the statistical foundations of the preeminent model used in this field: sequential Monte Carlo (SMC) algorithms.

2 The Presence of Duplicate Maps is Expected and a Sign that the SMC Algorithm is Working Correctly

Duplicate maps in a Sequential Monte Carlo (SMC) redistricting ensemble are not a defect. They are an essential and expected feature of the algorithm. Think of it like a survey where some respondents are given greater “weight” because their responses are needed to reflect the population you’re trying to represent. In survey research, when you give more weight to certain respondents (e.g., underrepresented or hard to contact groups), you will see the same respondent appear multiple times in a weighted calculation. That is not only intentional and correct, it is an important part of proper survey research methods.¹ SMC works the same way. As the algorithm progresses, it “resamples” (copies) maps that satisfy the legal constraints and meet the desired quality metrics. The highest-quality maps accrue higher weights and naturally appear multiple times in the final set, so duplication is a built-in mechanism for controlling statistical validity and computational efficiency. When a plan performs particularly well, showing excellent population balance, compactness, and respect for boundaries, it is assigned a higher probability and naturally appears more often in the resampling process. This is how the algorithm efficiently explores the enormous space of possible plans while maintaining statistical validity and computational efficiency.

Duplication in this context is therefore a feature, not a flaw. It means the algorithm has identified some maps as especially representative of how districts are likely to form under neutral criteria, and it gives those maps more weight to reflect that importance. It’s similar to how survey researchers give more weight to certain respondents so the overall survey better mirrors the population.

Every authoritative source on SMC methods confirms this behavior. McCartan and Imai (2023), in their peer-reviewed description of the *redist* package, explain that SMC “resamples” maps with higher importance weights, leading to duplication among high-quality

¹I am the associate principle investigator of the Cooperative Election Study, the largest recurring academic survey of the American public. <https://cces.gov.harvard.edu/people>

plans.² Academic research on the topic describe the same principle. Del Moral, Doucet, and Jasra (2012) state, “These resampling steps are crucial and, without them, it is impossible to obtain time uniform convergence results for SMC estimates.”³ In other words, duplicates are not a sign of failure. They are proof that the algorithm is properly concentrating on the most representative regions of the map space.

Dr. Chen may highlight it to be sensational, but even a sequence of one hundred or so identical maps within an ensemble of 50,000 is statistically trivial, representing just 0.2 percent of the total sample. Even if roughly 70 percent of the 50,000 simulated maps contain a duplicate, that result remains fully acceptable when standard diagnostic measures show no evidence of poor performance. When the feasible space of valid plans is smaller, the algorithm will necessarily spend more weight on those fewer configurations that best satisfy the legal and geometric constraints. Far from indicating bias, this shows that the algorithm is operating efficiently, focusing where the most probable and legally compliant maps reside.

Furthermore, there are statistical tests to determine if these repetitions are problematic. In short, they are not. I examined every relevant convergence and diversity diagnostic available in the *redist* program: effective sample size, plan acceptance rates, standard deviation of the sample weights, resampling rate, plan diversity ranges, and bottleneck detection. None of these metrics indicated any instability, concentration issues, or lack of plan diversity. These tests confirm that the algorithm explored the available map space broadly and without distortion. In other words, the ensemble contained appropriate diversity even though many high-probability maps appeared more than once. The goal is not to avoid duplication at all costs, but to ensure that the set of unique maps faithfully represents the overall distribution of valid redistricting plans. On that measure, my ensemble performs exactly as expected.

By contrast, there are no such diagnostic tools, or at least none that have ever been

²McCartan, Cory, and Kosuke Imai. “Sequential Monte Carlo for sampling balanced and compact redistricting plans.” *The Annals of Applied Statistics* 17, no. 4 (2023): 3300-3323.

³Del Moral, Pierre, Arnaud Doucet, and Ajay Jasra. “On adaptive resampling strategies for sequential Monte Carlo methods.” (2012): 252-278.

See also, Doucet, Arnaud, and Anthony Lee. “Sequential monte carlo methods.” In *Handbook of graphical models*, pp. 165-188. CRC Press, 2018.

disclosed, for Dr. Chen’s proprietary algorithm. There is no published documentation, no peer-reviewed validation, and no evidence that his model has been tested for convergence, diversity, or representativeness. It is therefore impossible to know whether his algorithm explores the space of possible maps appropriately or whether it is stuck in a narrow, biased region of outcomes. The *redist* package, by contrast, has been openly developed, extensively validated in academic research, and used in multiple court cases. Every SMC ensemble used in redistricting litigation has included some level of duplication, and courts have consistently credited these methods as reliable and scientifically sound.

Dr. Chen’s misunderstanding of duplication is especially ironic for several reasons.

First, it is difficult to understand how Dr. Chen, who has written and deployed his own redistricting algorithm, could be unaware of such a basic property of Sequential Monte Carlo methods. Every SMC-based simulation framework, including those used and validated in published academic research, produces duplicate maps as part of the resampling process. The fact that he presents this as a fatal flaw raises serious questions about whether he truly misunderstands the method or is deliberately attempting to mislead the Court about how modern, peer-reviewed redistricting algorithms actually function.

The second is a simple numbers game. If Dr. Chen is concerned about my simulations lacking a sufficient number of unique maps, even if one were to remove the duplicates (which I do not advise, since it undermines the statistical integrity of the sample), there nevertheless remain more than 14,000 unique maps, roughly 40% more than he produced.

Finally, he confuses the natural, mathematically necessary replication of high-probability maps in an SMC ensemble with the far more serious problem of systematic geographic bias evident in his own simulations. In my ensemble, duplication reflects the correct statistical weighting of neutral, well-performing plans. In his, nearly every “unique” map is substantively identical in another way. The algorithm repeatedly produces a district centered in the same region of northern Salt Lake County. The fact that his maps differ by a few census blocks or precincts does not make them meaningfully distinct. True bias is not about dupli-

cates, it's about where the algorithm repeatedly goes in the map space. His method keeps returning to the same partisan-favorable region of the county, producing what amounts to thousands of near-identical Democratic-leaning districts. Note that in my October 22nd, reply report, Figure 7 showed that his algorithm returns to that same portion of the county more than 90% of the time.⁴ In short, Dr. Chen attacks the feature of my model that ensures statistical validity, while his own algorithm displays the very defect, systematic, one-sided bias, that courts and scholars view as the clearest warning sign of a flawed simulation.

The *redist* algorithm is conducting a representative survey of Utahns by giving different people survey “weights” to ensure the sample is representative of the broader population. Dr. Chen’s algorithm is trying to conduct a representative survey of Utah by calling 10,000 different people, but only those whose phone numbers begin with (801).

⁴Dr. Chen attempts to explain this by saying that Draper and Bluffdale at the southern end of the county extend partly into Utah County, and therefore it would not make sense to include them in a Salt Lake County centric district. This fails on two counts. First, his Salt Lake Centric districts often extend northward and split Davis County, indicating that the algorithm could likewise do the same in a southward direction into Utah County. Second, this does not explain why the algorithm would also avoid Herriman, Riverton, South Jordan, Sandy, and Granite (all cities in the southern half of the valley that are entirely within Salt Lake County) with roughly the same frequency. Finally, Bluffdale’s extension into Utah County contains zero population.

3 Removal of Duplicates Does not Alter My Conclusions

Eliminating duplicates after sampling artificially “flattens” the underlying distribution and destroys the very weighting that makes the ensemble statistically sound. For this reason, researchers do not advise removing duplicates when evaluating partisan or geometric properties of smc ensembles.

Nevertheless, to demonstrate that this critique has no bearing on the substantive results, I conducted a secondary analysis using only the 14,668 unique maps. As shown in Figures 1 through 4, the results remain consistent across every key metric of fairness and representativeness. The enacted 2025 congressional plan is not an outlier under any measure. Its ranked marginal deviation, least republican vote share, standard deviation of the vote shares, and efficiency gap all fall well within the expected range of outcomes generated by the unique subset of simulations.

This analysis underscores an important point. The conclusions of my report do not depend on the number of duplicate maps but on the statistical properties of the ensemble as a whole. The SMC process assigns probability weights to each plan based on how well it satisfies the neutral redistricting criteria, ensuring that even when duplicates occur, the ensemble correctly represents the likelihood of each configuration. Removing duplicates does not change the substantive pattern of results. It merely discards information about how representative each map is in the distribution.

In short, duplication is a feature of the algorithm’s design, not a flaw. The removal of duplicates, which I explicitly advise against in line with the scholarly literature, has no meaningful effect on the findings presented in this report.

Figure 1

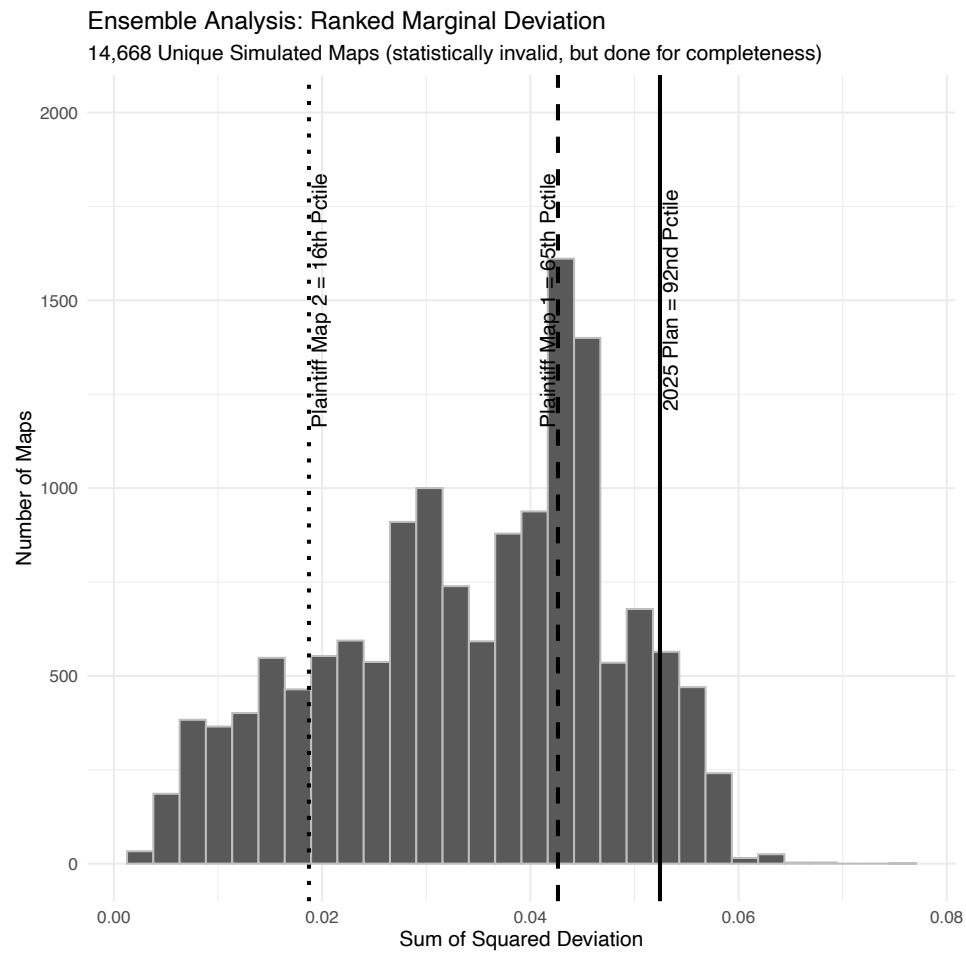


Figure 2

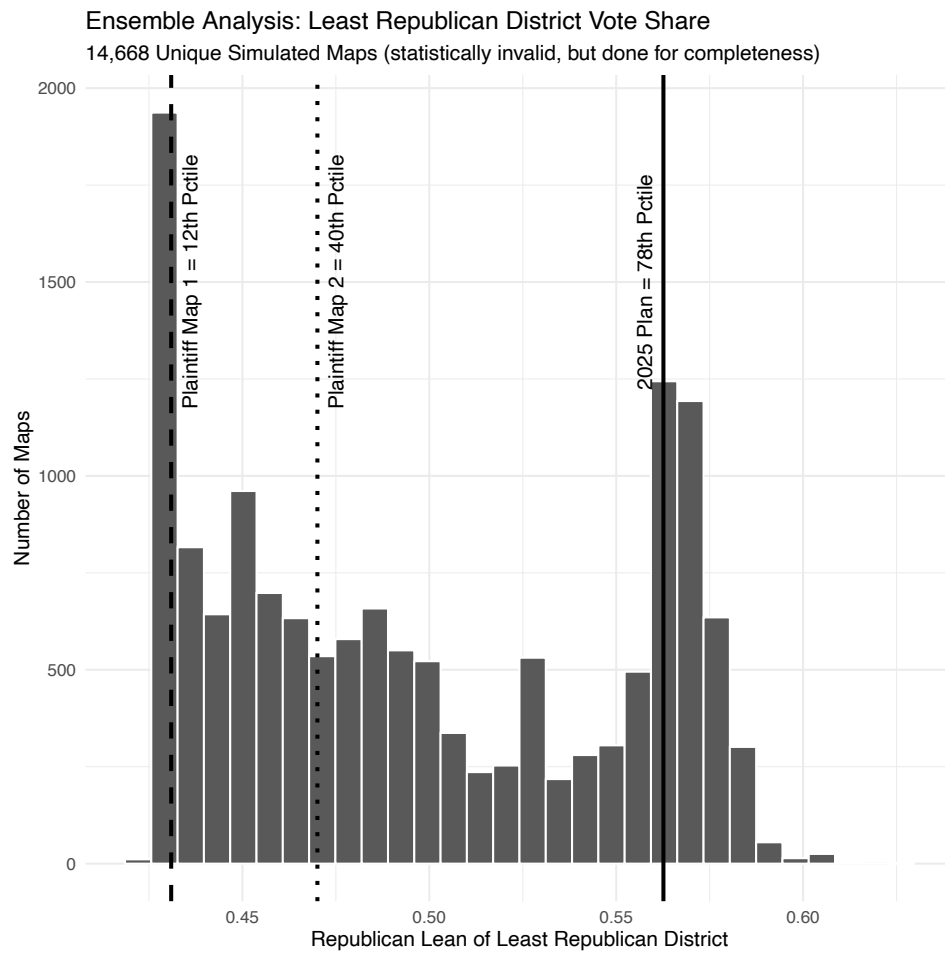


Figure 3

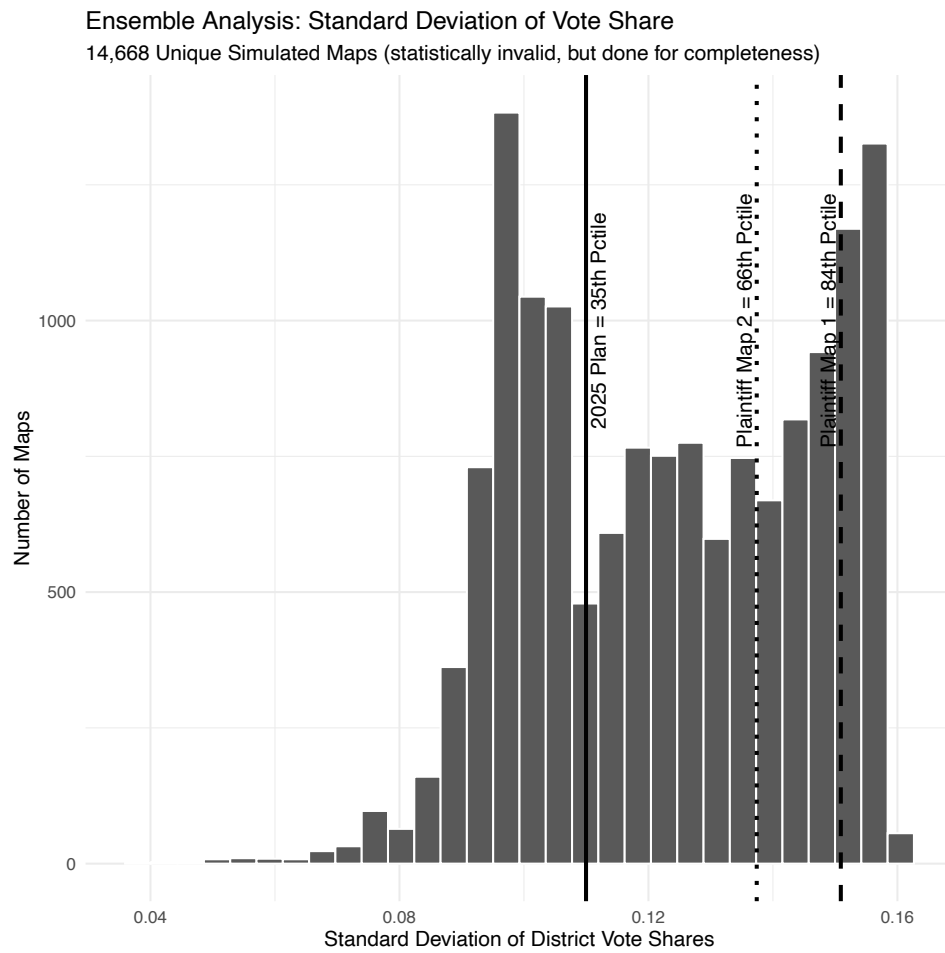
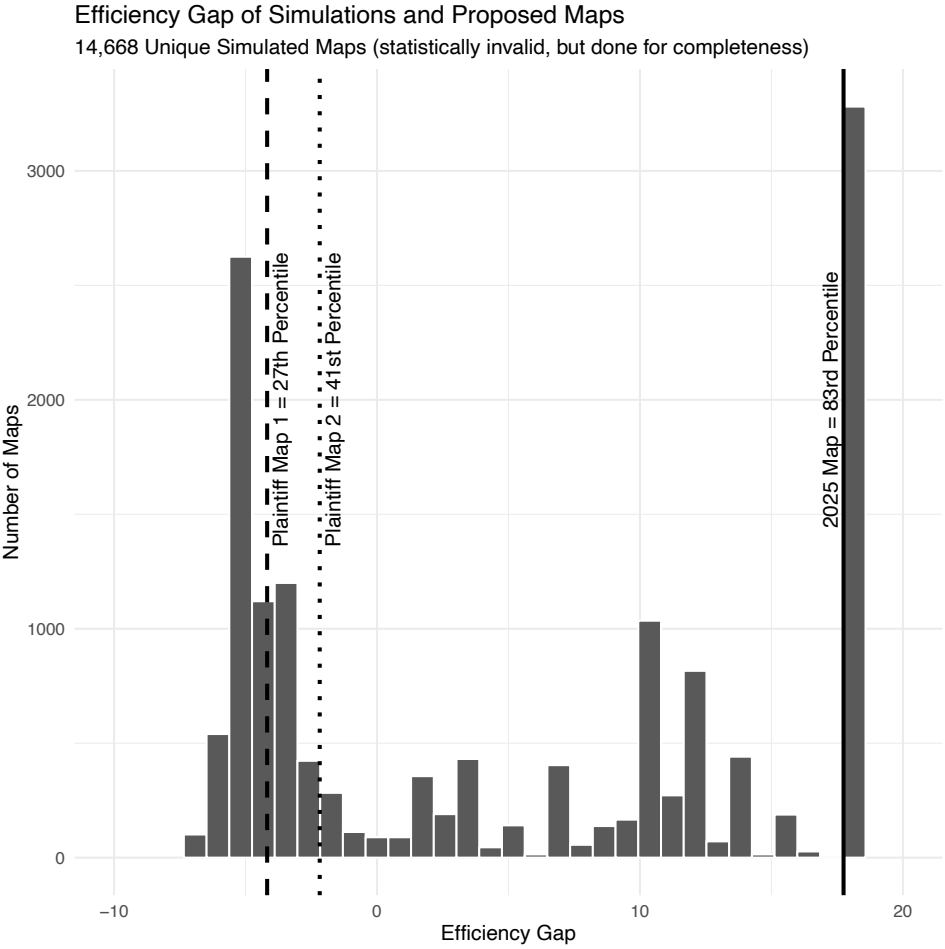


Figure 4



4 Summary

In summary of all of the reports that have been submitted in the last 7 days, the 2025 Plan is the only plan that has met the following tests:

- Partisan Bias Test
- Mean-Median Test
- Mean-Median Test (Warshaw-preferred method)
- Ranked Marginal Deviation - 50,000 simulations
- Ranked Marginal Deviation - partisan symmetry passing maps
- Ranked Marginal Deviation - county/municipal/compactness/SLCo restricted set
- Ranked Marginal Deviation - unique simulation maps only
- Least Republican Vote Share - 50,000 simulations
- Least Republican Vote Share - partisan symmetry passing maps
- Least Republican Vote Share - county/municipal/compactness/SLCo restricted set
- Least Republican Vote Share - unique simulation maps only
- Standard Deviation of Vote Share - 50,000 simulations
- Standard Deviation of Vote Share - partisan symmetry passing maps
- Standard Deviation of Vote Share - county/municipal/compactness/SLCo restricted set
- Standard Deviation of Vote Share - unique simulation maps only
- Efficiency Gap - 50,000 simulations
- Efficiency Gap - partisan symmetry passing maps
- Efficiency Gap - county/municipal/compactness/SLCo restricted set
- Efficiency Gap - unique simulation maps only

Michael Barber

Signed: _____

Dated: October 23, 2025